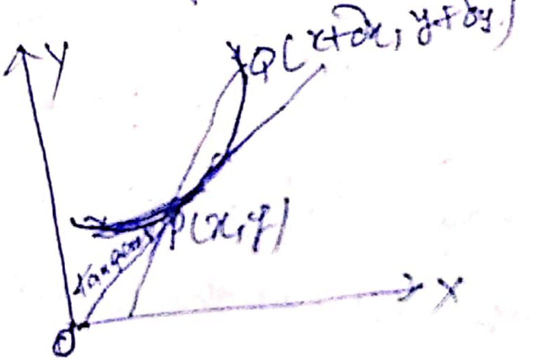
Lectured By DR. H. K. Yadav.
Deptt. of Mathematics.
Marwari College, Darbhanga.
B.Sc - part - III (H), Paper - III, 4
Tangents and Normals. Date - 03 - 06 - 2020

Q.1 Find the eqn to the tangent to the curve $y = f(x)$ at (x, y) is $Y - y = \frac{dy}{dx}(X - x)$ and that to the curve $f(x, y) = 0$ is
$(X - x) f_x + (Y - y) f_y = 0$

Soln:



Let P and Q be two points on the curve. By the tangent to the curve at P we mean the straight line which is the limiting position to which secant PQ tends as Q tends to P travelling along the curve.

Let the co-ordinates of the points P and Q be respectively (x, y) and $(x+\delta x, y+\delta y)$.

Let the eqn to the curve be $y = f(x)$
Then, $y + \delta y = f(x+\delta x)$ when $(x+\delta x, y+\delta y)$ is on the curve.
$$\therefore \frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$$
Let (X, Y) be the current co-ordinates.
Then the eqn to the straight line PQ Joining the points $P(x, y)$ and $Q(x+\delta x, y+\delta y)$ is

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$$\frac{Y-y}{X-x} = \frac{(y+\delta y)-y}{(x+\delta x)-x} \Rightarrow \frac{Y-y}{X-x} = \frac{\delta y}{\delta x} = \frac{(y+\delta y)-f(x)}{\delta x}$$

Now, let Q tend to P. Then $\delta x \to 0$ and the straight line PQ will become the tangent to the curve at $P(x, y)$

$$\therefore \frac{Y-y}{X-x} = \lim_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x}$$

$$= f'(x) = \frac{dy}{dx}$$

Hence, the eqn to the tangent to the curve $y = f(x)$ at $P(x, y)$ is

$$Y-y = \frac{dy}{dx}(X-x) \quad \text{———} \quad (I)$$

Now, if the eqn of the curve be given in the form $f(x, y) = 0$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

Substituting the value of $\frac{dy}{dx}$ in (I), we get

$$Y-y = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}(X-x)$$

$$\Rightarrow (X-x)\frac{\partial f}{\partial x} + (Y-y)\frac{\partial f}{\partial y} = 0$$

$$\Rightarrow (X-x)f_x + (Y-y)f_y = 0$$

which is the eqn of the tangent to the curve $f(x, y) = 0$ at $P(x, y)$

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